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A Study of Heat Transfer for Two Layered Composite Inclined Plate Crotch Absorbers

1 INTRODUCTION

Since a copper plate (Z=29) absorbs most of the photon energy very near the surface, the temperature of the surface becomes very high despite of having a high thermal conductivity. On the other hand, a beryllium plate(Z=4) can diffuse the intense radiation throughout the depth of its plate by allowing photons to penetrate, but has a low thermal conductivity (about half of that of a copper). As an effort to combine both merits of Be and Cu , a Be-Cu composite absorber[1] was developed and has been successfully used in CESR. They analyzed the heat transfer problem numerically for the case of a vertically located Be-Cu composite cylinder which results in symmetry with respect to the center of photon beam and allows them to consider only half a domain. In this note, an inclined absorber with two layered metal plates is considered and a full domain solution is sought to study the asymmetric heating due to the inclined photon beam penetration heating. An analytical solution for heat transfer is obtained for a full domain using the Fourier integral transformation and of particular interests are the effects of different thickness ratios of two materials and different inclination angles.

2 ANALYSIS

Consider a two layered composite inclined plate with angle θ with respect to the incident photon beam (see Figure 1). Assuming that conductivities and absorption coefficients for each layer are different but constant which are k_1, k_2 and α_1, α_2, respectively, the following governing equations are obtained for each layer (refer to [2] for the single homogeneous material case);

$$\nabla^2 T_1 = C_{A1} exp(-C_{A2} x_1) exp(-\frac{(y+\lambda_1)^2}{4\rho}) \tag{1}$$

$$\nabla^2 T_2 = C_{B1} exp(-C_{B2} x_2) exp(-\frac{(y+\lambda_2)^2}{4\rho}) \tag{2}$$

where

$$C_{A1} = -5425 \frac{\alpha_1 {a_t}^2}{k_1 T_\infty} \frac{E^4 BI}{l^2}$$

$$C_{A2} = \frac{\alpha_1 a_t}{sin\theta}$$

$$\lambda_1 = -x_1 cot\theta$$

$$C_{B1} = -5425 \frac{\alpha_2 {a_t}^2}{k_2 T_\infty} \frac{E^4 BI}{l^2} exp(-\alpha_1 \frac{a_1}{sin\theta})$$

$$C_{B2} = \frac{\alpha_2 a_t}{sin\theta}$$

$$\lambda_2 = -(x_2 + \frac{a_1}{a_t}) \cdot cot\theta$$

$$\frac{1}{4\rho} = \left(\frac{sin\theta a_t \gamma}{0.608 l \sqrt{2}} \right)^2$$

where E, I, B, l and γ are the positron energy in GeV, the positron beam current in mA, the magetic field in T, the tangential length from the source to the plate in m and $\gamma = \frac{E}{m_o c^2} = 1957E$, respectively.

In the above equations, dimensionless variables x, y and T are defined as

$$x_1 = \frac{x_1^*}{a_t}, \quad x_2 = \frac{x_2^*}{a_t}, \quad y = \frac{y^*}{a_t}$$

$$T = \frac{T^* - T_\infty}{T_\infty}$$

where $*$ denotes the dimensional quantity and a_t is the total thickness of the composite plate($= a_1 + a_2$)

At the interface at $x_1 = \frac{a_1}{a_t}$ and $x_2 = 0$, the continuity requirement of the temperature and the heat flux gives the following boundary conditions;

$$T_1 = T_2 \tag{3}$$

$$\frac{\partial T_1}{\partial x_1} = \frac{k_2}{k_1} \frac{\partial T_2}{\partial x_2}$$

Boundary conditions on other surfaces are given in figure (1).

Applying the Fourier transformation defined by

$$\bar{T}(x,z) = \int_{-\infty}^{\infty} T(x,y) e^{-iyz} dy \tag{4}$$

the following ordinary differential equations are obtained:

$$\frac{d^2\bar{T}_1}{dx_1^2} + (iz)^2\bar{T}_1 = C_{A1} exp(-C_{A2}x_1) 2\sqrt{(\pi\rho)} exp(-\rho z^2 + i\lambda_1 z) \tag{5}$$

$$\frac{d^2\bar{T}_2}{dx_2^2} + (iz)^2\bar{T}_2 = C_{B1} exp(-C_{B2}x_2) 2\sqrt{(\pi\rho)} exp(-\rho z^2 + i\lambda_2 z) \tag{6}$$

The solutions for the above O.D.E. may be written as:

$$\bar{T}_1 = A_1 exp(zx_1) + A_2 exp(-zx_1) + D_A exp(-E_A x_1) \tag{7}$$

$$\bar{T}_2 = B_1 exp(zx_2) + B_2 exp(-zx_2) + D_B exp(-E_B x_2)$$

where

$$D_A = \frac{C_{A1} 2\sqrt{\pi\rho} exp(-\rho z^2)}{(C_{A2} + izcot\theta)^2 - z^2}, \; E_A = C_{at} + izcot\theta$$

$$D_B = \frac{C_{B1} 2\sqrt{\pi\rho} exp(-\rho z^2) exp(-i\frac{a_1}{a_t} zcot\theta)}{(C_{B2} + izcot\theta)^2 - z^2}, \; E_B = C_{B2} + izcot\theta$$

and A_1, A_2 ,B_1 and B_2 are unknowns to be determined from boundary conditions.

Applying the Fourier transformation to boundary conditions yields :

$$\frac{d\bar{T}_1}{dx_1} = 0 \;\; at \; x_1 = 0 \tag{8}$$

$$\bar{T}_1 = \bar{T}_2 \;\; at \; x_1 = \frac{a_1}{a_t} \; and \; x_2 = 0$$

$$\frac{d\bar{T}_1}{dx_1} = \frac{k_2}{k_1}\frac{d\bar{T}_2}{dx_2} \;\; at \; x_1 = \frac{a_1}{a_t} \; and \; x_2 = 0$$

$$\frac{d\bar{T}_2}{dx_2} = -\frac{ha_t}{k_2}\bar{T}_2 \;\; at \; x_2 = \frac{a_2}{a_t}$$

where h is the heat transfer coefficient.

Substituting equation (7) into equation (8), a linear set of algebraic equations is obtained:

$$A_1 z - A_2 z = g_1 \tag{9}$$

$$A_1 z exp(z\frac{a_1}{a_t}) - A_2 z exp(-z\frac{a_1}{a_t}) - k_r B_1 z + k_r B_2 z = g_2$$

$$A_1 exp(z\frac{a_1}{a_t}) + A_2 exp(-z\frac{a_1}{a_t}) - B_1 - B_2 = g_3$$

$$B_1 exp(z\frac{a_2}{a_t})(z - Bi) - B_2 exp(-z\frac{a_2}{a_t})(z + Bi) = g_4$$

where

$$g_1 = D_A E_A$$

$$g_2 = D_A E_A exp(-E_A\frac{a_1}{a_t}) - k_r D_B E_B$$

$$g_3 = D_B - D_A exp(-E_A\frac{a_1}{a_t})$$

$$g_4 = D_B E_B exp(-E_B\frac{a_2}{a_t}) + Bi D_B exp(-E_B\frac{a_2}{a_t})$$

$$k_r = \frac{k_2}{k_1}, \quad Bi = -\frac{h a_t}{k_2}$$

and final results for A_1, A_2, B_1 and B_2 are written in the appendix.

It is necessary to invert $\bar{T}$ from the transformed coordinate z to the physical coordinate y. Since $\int_{-\infty}^{\infty} T(x,y)dy$ is finite, the Fourier inversion path in the complex z plane can be taken along the real axis. The inversion of $\bar{T}$ yields:

$$T(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{T}(x,z)e^{izy}dz \tag{10}$$

The above integration was carried out numerically(the effects of the values of the upper and lower limits of integration and the size of the interval dz were tested).

3 RESULTS AND DISCUSSIONS

An analytical solution has been obtained for heat transfer of a two layer composite inclined plate absorber. When two layers are taken as same materials, the present solution should represent the single material absorber case for which a series solution was obtained in [2]. Figure (2) shows the comparison of the present solution with the previous series solution for single material absorber case. Taking both plate 1 and 2 to be beryllium plates inclined with 50 degree, the present calculation for a composite absorber shows very good agreement with the previous solution for the single material absorber, which confirms both solutions.

Table (1) shows the parameters and the property data used for Be and Cu plates.

Figure (3-1) shows the temperature distributions on the vacuum surface and interface between beryllium-copper composite plates (Thickness: Be=2.5 mm and Cu=20 mm). Solid lines designate temperatures on vacuum surface ($x_1 = 0$) and dashed lines are for the interface ($x_1 = \frac{a_1}{a_t}$ and $x_2 = 0$). Note that the maximum temperature occurs at the interface (dashed curves) (this also occured in [1]). For single material absorbers, the maximum always occurs at the vacuum surface (refer to [2]). This is due to different absorption coefficients of each plate, e.g., $\alpha_1 = 53.02[1/m]$ was used for beryllium and $\alpha_2 = 33560[1/m]$ for copper. Small value of the coefficient for Be plate allows photons to distribute throughout the plate and the portion of the incident radiation passes through the layer. Penetrated photons through Be are then absorbed near the surface of the second layer of copper since it has much larger absorption coefficient than Be, which results in the maximum temperature at the interface of Be-Cu. For a vertical case of $\theta = 90$, symmetric temperature distributions were obtained for both plates with respect to the center line at $y = 0$. As the angle decreases, temperatures decrease because the peak power density decreases. It is noted that the y-location where the maximum occurs shifts upward as the angle decreases. These locations correspond to the points where pentrated photons through the center of Be plate ($x_1 = 0$ and $y = 0$) deposit on the interface ($x_2 = 0$). Therefore, as the plate inclines further, those points should occur at further positive y axis. This effect can be shown more clearly in figures (3-2) and (3-3) for thicker Be plates. Figures (3-2) and (3-3) show the cases of 5 mm thick Be and 20 mm thick Cu composite plate and 7.5 mm Be and 20 mm Cu composite, respectively. Note in Fig.(3-2) and (3-3) that for the cases of $\theta = 10°$, the vacuum surface has a larger peak temperature than the interface (recall that for the cases of $\theta > 10°$ of 5 mm and 7.5 mm Be and all cases of 2.5 mm Be, the interface has a larger peak temperature than the vacuum surface). This can be explained as follows. The inclination of the plate causes photon path to become larger, which signifies that heating is more distributed and at the same time, larger portion of the incident radiation is absorbed in the Be layer, correspondingly, the Be surface (vacuum surface) may have a larger peak than the Cu surface (interface). It is also shown in Fig. (3-3) that the vacuum surface temperature at $\theta = 10°$ become larger that that of $\theta = 30°$, which can be explained similarly. For cases of $\theta > 10°$, the effect of distributed heating is dominant and the absorbed portion of the incident radiation in Be layer is not significant, which causes both peak temperatures at the vacuum surface and interface to become reduced and the interface has a larger temperature that the vaccum surface.

Figure (4) shows the temperature distributions of the water cooled surface for three different thickness ratios of Be and Cu. Solid curves are for the case of 2.5 mm Be and 20 mm Cu, dashed ones for 5 mm Be and 20 mm Cu and dotted ones for 7.5 mm Be and 20 mm Cu. For all three different thicknesses, cases of $\theta \geq 30°$ result in almost same peak temperatures which are shifted in the positive y direction as inclined further(recall that peak temperatures of the vacuum and the interface became reduced as the angle decreased). This is due to good diffusion of the same total energy through 20 mm thick Cu plates. The calculations for different thicknesses of Cu layers but same thickness of Be layer resulted in larger peak temperatures for thinner plates(not included in this note). But, cases of $\theta = 10°$ show some reduction of peak temperatures ranging 2-10 %, which shows the advantage of distributed heating through Be layer. It is also shown that as the thichness of Be becomes larger, the peak temperature becomes smaller. Note that for the cases of $\theta = 10°$ for 5 mm and 7.5 mm Be, the peak temperatures become less than boiling temperature at 5 atm.

Figure (5) shows the isotherm lines inside the Be-Cu composite plate (5mm Be and 20 mm Cu case with 10 degree angle). One can see the distinctive effect of the penetration through Be layer and discontinuity of temperature gradients at the interface due to having different thermal conductivities.

In summary, an analytical solution was obtained for heat transfer of two layered composite inclined crotch absorbers and it was shown that Be-Cu composite absorber would lower vacuum and interface maximum temperatures significantly and also lower the peak temperatures of water cooled surface (for the cases of $\theta = 10°$ of 5 mm and 7.5 mm Be, the peak temperatures are below the boiling temperature.).

REFERENCES

[1] D.M. Mills, D.H. Bilderback and B.W. Batterman, "Thermal Design of Synchrotron Radiation Ports at CESR", IEEE Transactions on Nuclear Science, Vol. NS-26, No.3, pp. 3854-3856, 1979

[2] M. Choi, "A Note on Thermal Analysis for an Inclined Plate Crotch Absorber", ANL Light Source Note, June 1989

APPENDIX

$$A_1 * DN = exp(\tilde{a_2}z)\left(g_1 exp(-\tilde{a_1}z)((z + Bi \cdot k_r)cosh(\tilde{a_2}z) - (zk_r + Bi)sinh(\tilde{a_2}z))\right.$$

$$\left.-g_2(zcosh(\tilde{a_2}z) - Bi \cdot sinh(\tilde{a_2}z)) + g_3 k_r z(Bi \cdot cosh(\tilde{a_2}z) - zsinh(\tilde{a_2}z)) - g_4 k_r z\right)$$

$$A_2 * DN = exp(\tilde{a_2}z)\left(g_1 exp(\tilde{a_1}z)((z - Bi \cdot k_r)cosh(\tilde{a_2}z) + (zk_r - Bi)sinh(\tilde{a_2}z))\right.$$

$$\left.-g_2(zcosh(\tilde{a_2}z) - Bi \cdot sinh(\tilde{a_2}z)) + g_3 k_r z(Bi \cdot cosh(\tilde{a_2}z) - zsinh(\tilde{a_2}z)) - g_4 k_r z\right)$$

$$B_1 * DN = g_1(z + Bi) - g_2(z + Bi)cosh(\tilde{a_1}z) + g_3(z + Bi)zsinh(\tilde{a_1}z)$$

$$-g_4 exp(\tilde{a_2}z)z(sinh(\tilde{a_1}z) + k_r cosh(\tilde{a_1}z))$$

$$B_2 * DN = -exp(\tilde{a_2}z)\left(-g_1 exp(\tilde{a_2}z)(z - Bi) + g_2 exp(\tilde{a_2}z)(z - Bi)cosh(\tilde{a_1}z)\right.$$

$$\left.-g_3 exp(\tilde{a_2}z)(z - Bi)zsinh(\tilde{a_1}z) - g_4 z(sinh(\tilde{a_1}z) - k_r cosh(\tilde{a_1}z))\right)$$

$$DN = (z + Bi)z(k_r cosh(\tilde{a_1}z) - sinh(\tilde{a_1}z))$$

$$-exp(2\tilde{a_2}z)(z - Bi)z(sinh(\tilde{a_1}z) + k_r cosh(\tilde{a_1}z))$$

where

$$\tilde{a_1} = \frac{a_1}{a_t}$$

$$\tilde{a_2} = \frac{a_2}{a_t}$$

Table (1) Parameters and Property Data Used in This Analysis

E	B	I	l
7 GeV	0.6 T	300 mA	1.75 m
beryllium		*copper*	
k_1, W/m K	α_1, 1/m	k_2, W/m K	α_2, 1/m
161	53.02	386	33560

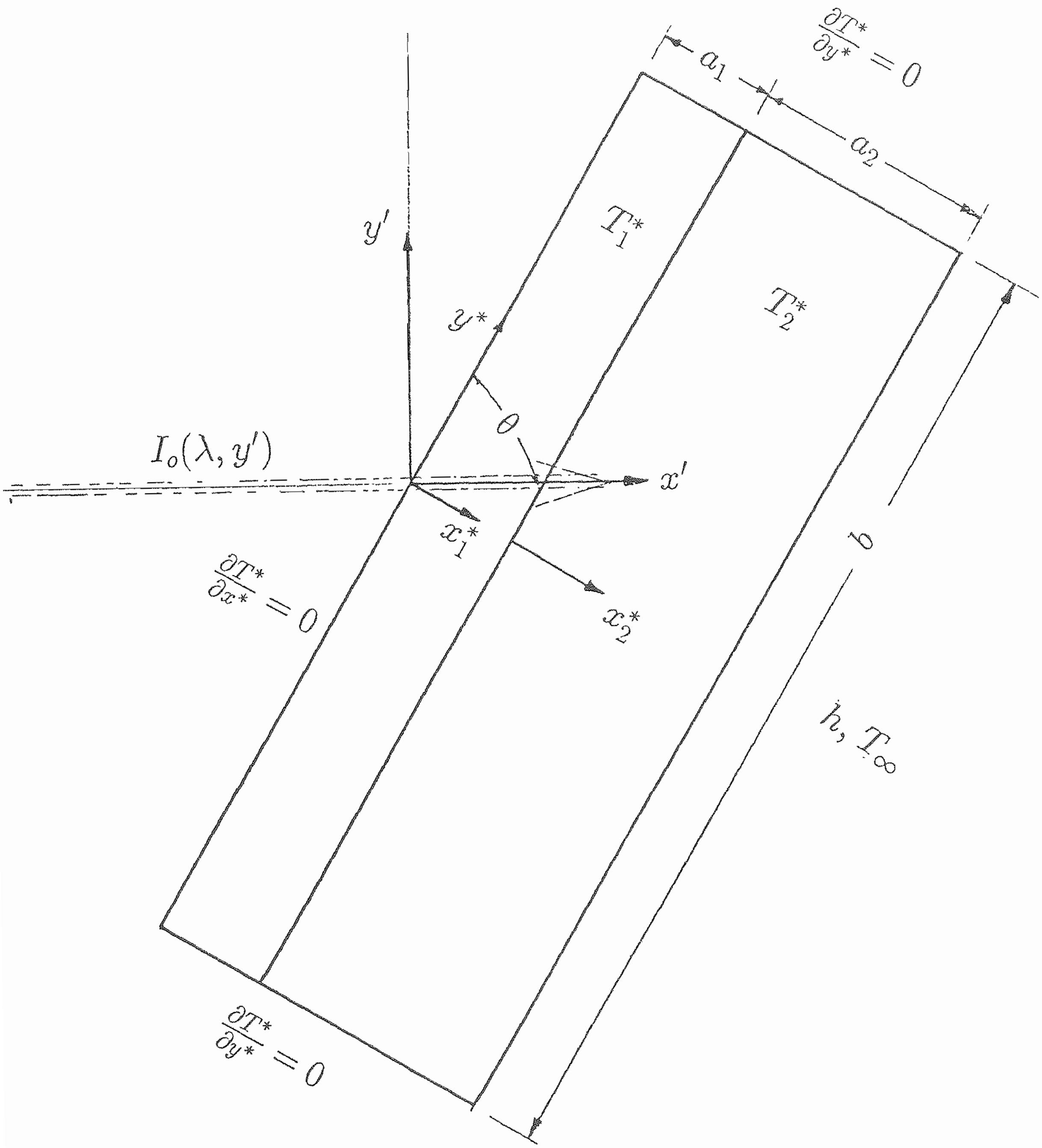

Figure (1 !) Geometry and Boundary Conditions for Two Layered Composite Inclined Plate Absorber

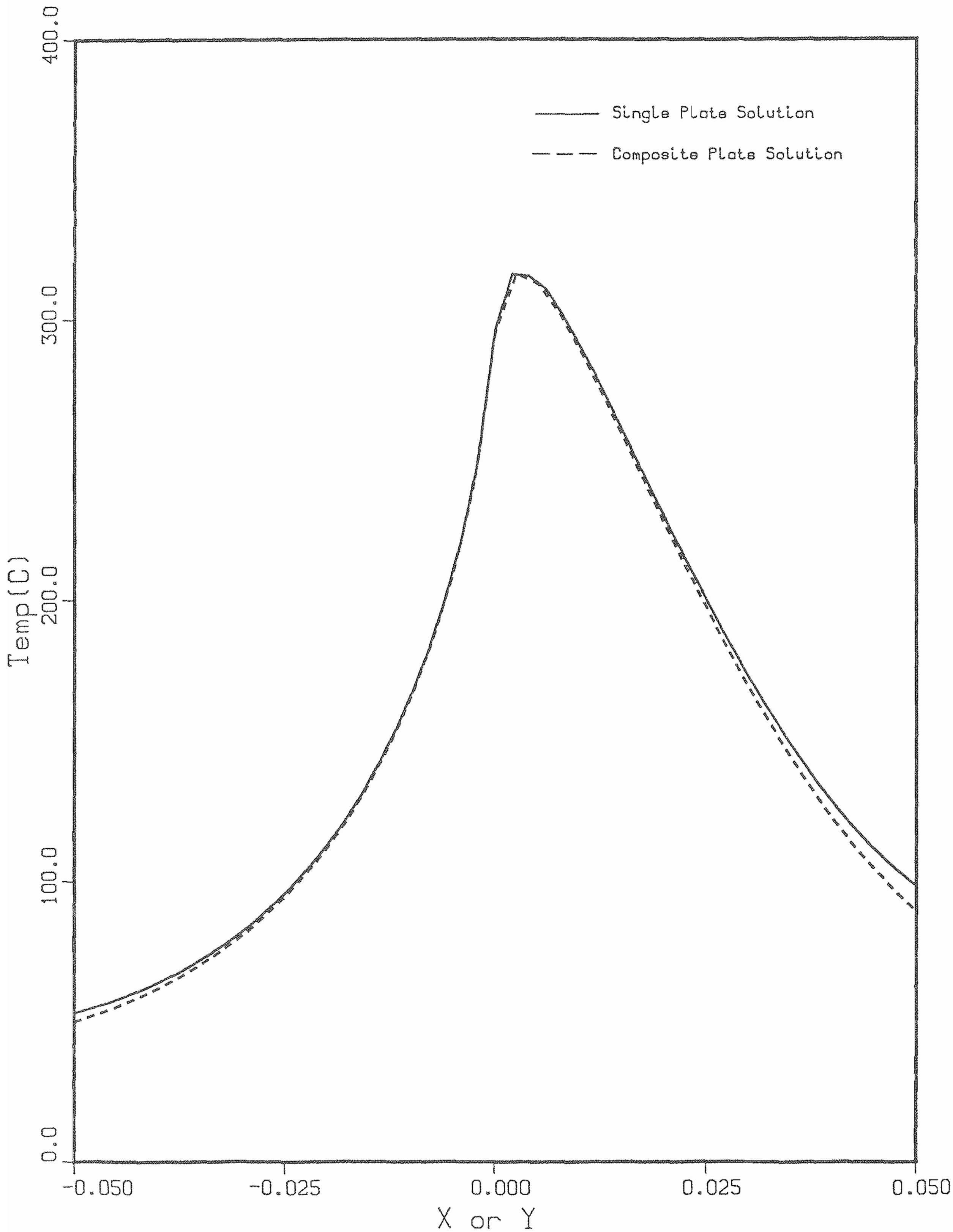

Fig. 2 Comparison of the Present Solution with the Previous Series Solution [2] ; $a_1 = 1cm$, $a_2 = 1cm$, $\theta = 30°$

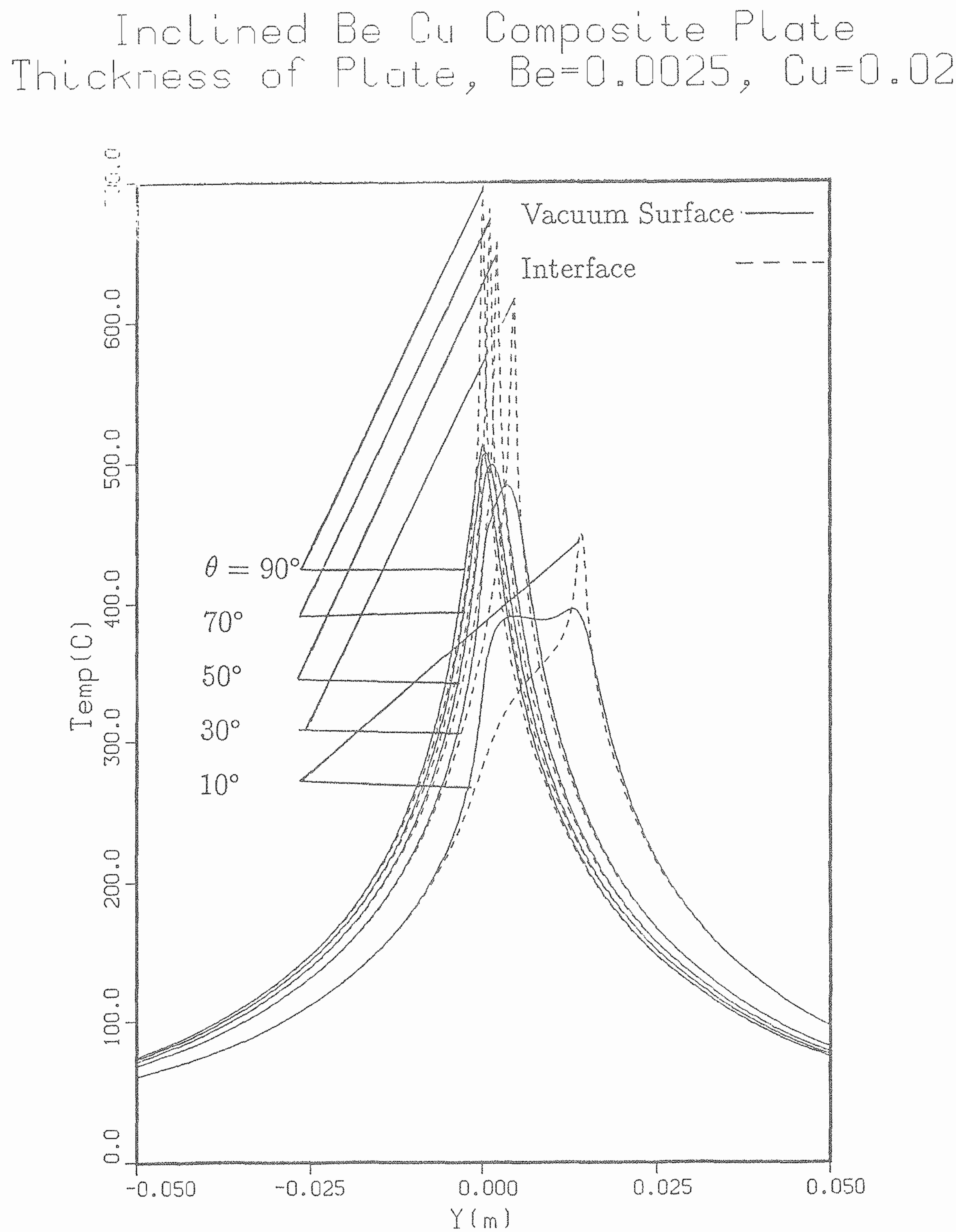

Fig.3-1 Temperature Distributions of the Vacuum Surface and Interface of Be-Cu Composite Plates Different Angles; 1=Be, 2=Cu, $a_1 = 2.5$ mm, $a_2 = 20$ mm

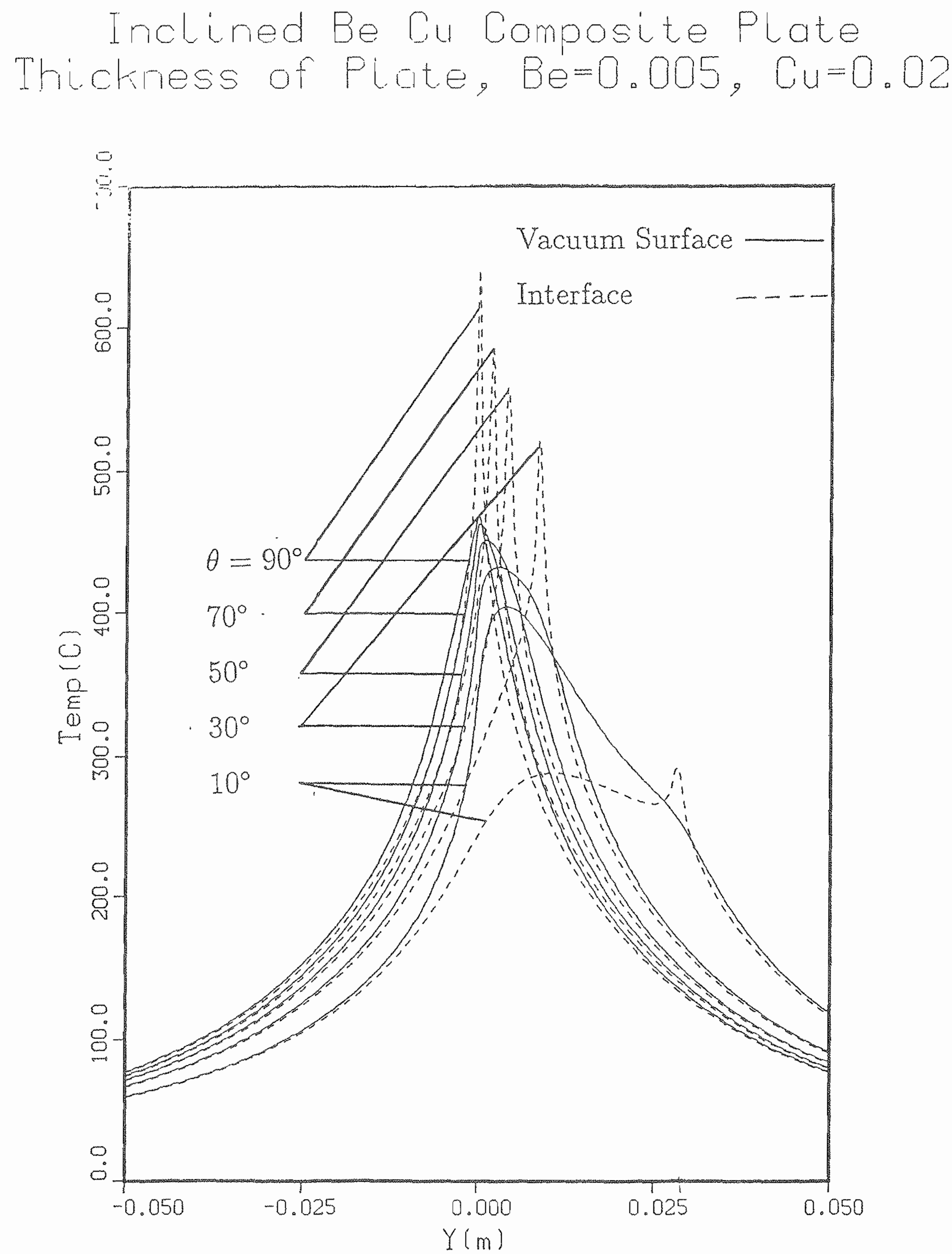

Fig.3-2 Temperature Distributions of the Vacuum Surface and Interface of Be-Cu Composite Plates for Different Angles; 1=Be, 2=Cu, $a_1 = 5$ mm, $a_2 = 20$ mm

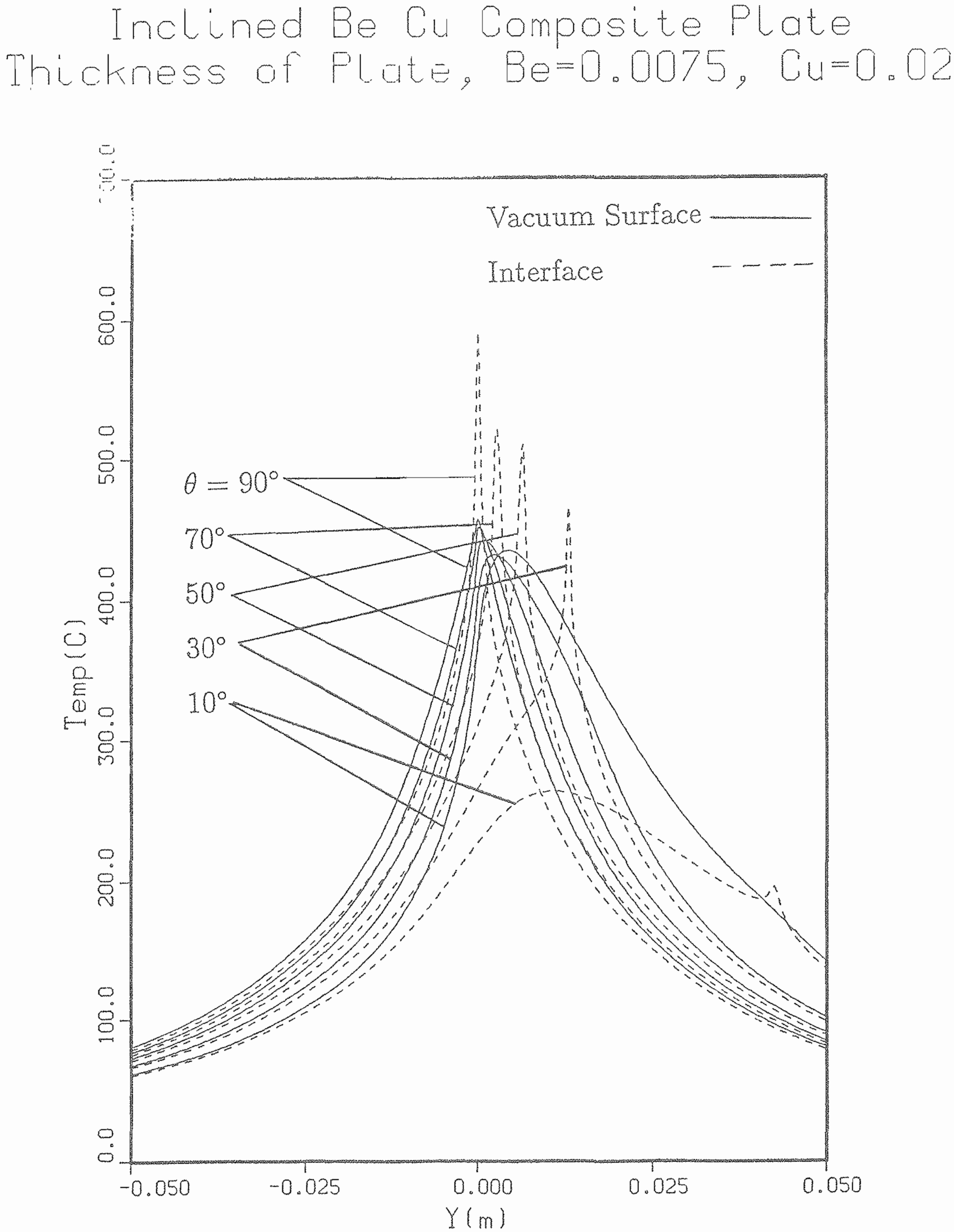

Fig.3-3 Temperature Distributions of the Vacuum Surface and Interface of Be-Cu Composite Plates for Different Angles; 1=Be, 2=Cu, $a_1 = 7.5$ mm, $a_2 = 20$ mm

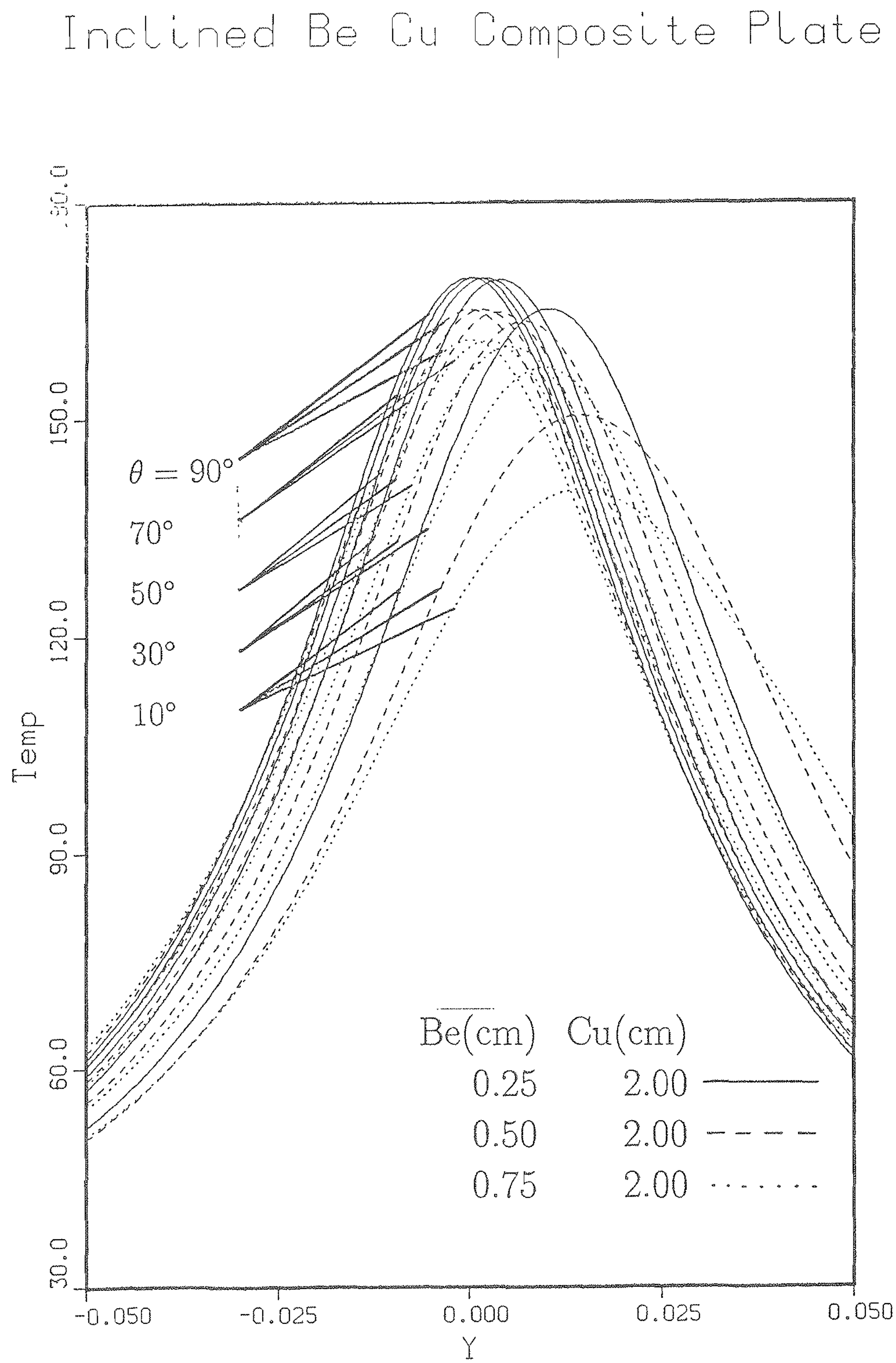

Fig.4 Temperature Distributions of the Water Cooled Surface for Different Thickness Ratios of Be and Cu and Different Angles

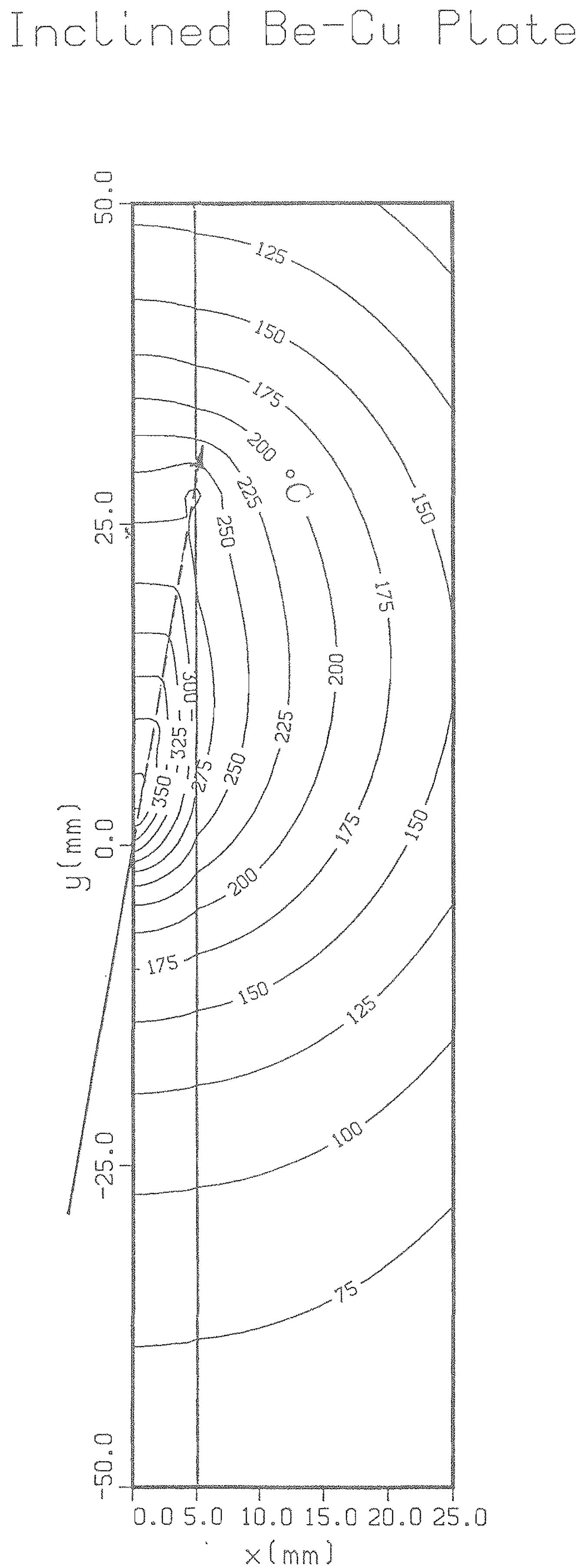

Fig.5 Isotherm Line Contours for Be-Cu Composite Inclined Absorber; 1=Be, 2=Cu, $a_1 = 0.5$ cm, $a_2 = 2$ cm, $\theta = 10°$